1. The colleague's claim is false, which can be shown by contradiction. If we run the experiment for a coin toss, it will be clear that

   $E(X^{٢})=E(X^{3})-E(X)$ does not hold. Here is the actual calculation:

   $$E(X)=\sum_{i=1}^{n} x_i P(X=x_i)$$
   $$E(X)=1^1(1/2)+2^1(1/2)=1.5$$

   $$E(X^2)=\sum_{i=1}^{n} x_i^2 P(X=x_i)$$
   $$E(X^2)=1^2(1/2)+2^2(1/2)=2.5$$

   $$E(X^3)=\sum_{i=1}^{n} x_i^3 P(X=x_i)$$
   $$E(X^3)=1^3(1/2)+2^3(1/2)=4.5$$

   $$2.5\neq 4.5-1.5$$
   $$2.5\neq 3.0$$

2. He is not correct. There are a couple of ways to show this. First, here is the general moment generating function:

   $$m_X(t)=E(e^{xt})$$

   The formula given by my colleague is as follows:

   $$m_{((x-a)^r)}(t)=E(e^{(((X-a)r)t)})$$

   First, on technical grounds, the substitution is incorrect. He seems to be performing the following substitution:

   $$x=(X-a)^r$$

   which would result in the following equation:

   $$m_{((X-a)^r)}(t)=E(e^{(((X-a)^r)t)})$$

   This is different than the equation specified by my colleague which had the exponent *(X – a)* being multiplied by *r* on the right side of the equation. In order for it to be a consistent substitution, *(X – a)* would need to be raised to *r*.

   However, beyond this (possibly typographic) error, there is a bigger problem. If you are calculating the *r*th moment around the center *a*, *r* and *a* should not appear in the moment generating function. That is the reason the moment generating function (listed above) exists.

3. I think this might be easier to explain using formulas than words:

$$
\begin{aligned}
&E(X)=\mu \\
&Skew(X)=E((X-\mu)^3) \\
&\qquad =E((X-\mu)[(X-\mu)(X-\mu)]) \\
&\qquad =E((X-\mu)(X^2-2X\mu+\mu^2)) \\
&\qquad =E(X^3-2X^2\mu-X^2\mu+X\mu^2+2X\mu^2+\mu^3) \\
&\qquad =E(X^3-3X^2\mu+3X\mu^2+\mu^3) \\
&\qquad =E(X^3)-3\mu E(X^2)+3\mu^2 E(X)+\mu^3 \\
&\qquad =E(X^3)-3\mu E(X^2)+3\mu^3+\mu^3 \\
&\qquad =E(X^3)-3\mu E(X^2)+4\mu^3
\end{aligned}
$$

We start with the formula for Skewness (which is the 3rd moment around the mean) and the knowledge that *mu* is equal to *E(X)*. After we have those 2 notions, we perform algebraic expansion on the formula.

Furthermore, we can calculate each expectation using the standard formulas for median, variance and skewness:

$$
\begin{aligned}
&E(X)=np=\mu \\
&Var(X)=E((X-\mu)^2)=np(1-p) \\
&Skew(X)=E((X-\mu)^3)=(1-2p)/\sqrt{np(1-p)}
\end{aligned}
$$

Substituting *np* for *mu* and expanding, anyone knowing college algebra should be able to derive formulas for *E(X^2)* and *E(X^3)*.

4. This is problem can be modeled using a binomial distribution. The primary question is what is *n* and what is *p*.

Here are the facts that we know from the problem description:

- Martian words each contain exactly 3 characters, no more no less.
- The Martian alphabet consists of 160 characters
- The Martian book contains 1,000,000 words, with the assumption that each word (3 character sequence) is independent, though this would not be the case in the real world.

From these basic facts we should be able to deduce what *n* and *p* are. First *n* which seems to be the most obvious. If we think of the book as the complete run of an experiment, with each word being one iteration of the experiment, *n* becomes obvious:

$$n=1{,}000{,}000$$

Calculating *p* is the trickier aspect of the problem. The first thing we need to calculate is number of possible strings. It should be understood, as is the case with English, that not every possible combination of 3 characters will be a word in the Martian language. However, since we do not have a Rosetta Stone for Martian, we have to consider every possibility combination of 3 symbols. The calculation is as follows:

$$word\ length = 3$$
$$alphabet = 160$$
$$potential\ words = 160^3 = 4{,}096{,}000$$

Again, it is unlikely that all of the 4,096,000 combinations will actually be words. However, they must all be included in the model. This is the denominator of our calculation for *p*. The numerator is the total number of words, which is 1,000,000. Calculating *p*, we get:

$$p = 1{,}000{,}000 / 4{,}096{,}000$$
$$p = 0.24414$$

Now that we have *p* and *n*, we can set up the model with $X=3$, which looks like this.

$$P(X=3) = 0.24414^3 (1-0.24414)^{999{,}997}$$

5.